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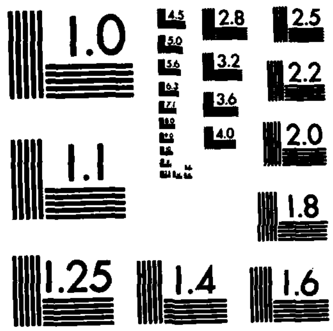
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SCATTERING COEFFICIENT ESTIMATION: AN EXAMINATION OF THE NARROW-BEAM APPROXIMATION

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ABSTRACT

One quick and easy way of estimating scattering coefficient is to assume that the antenna beam is confined in a finite cone with constant gain inside and the range to the antenna is constant for all the cells inside the illuminated area. Also the variation of the σ^o is assumed to be small enough inside the illuminated area so one can estimate the σ^o with only the power return measurements.

The above may not be true depending on the antenna gain shapes or the target characteristic. In this report, the so-called narrow-beam approximation is examined using a little more realistic antenna gain functions and range variation and also a specific σ^o variation function. The numerical integration shows that the narrow beam approximation can give as much as 3.6 dB error for normal incidence to the target with rapidly varying σ^o when the beamwidths are 9 deg wide. For the incidence angles of 10 deg or larger, the error is in the order of a dB or less depending on the beamwidths and target characteristics.

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1.0 BACKGROUND

Accurate estimation of scattering coefficient for an area-extensive target using the average return power is not a simple task because it involves an integral equation. Hence, it is usually simplified by using several approximations and assumptions. The purpose of this report is to investigate the validity of these commonly used approximations and to evaluate the possible error when these simplifying assumptions are used to calculate the scattering coefficient, for example antenna beamwidths and target scattering coefficient angular variations.

Axline [1] developed a matrix inversion technique to invert the integral equation for various cases of antenna gain shapes and σ^o variation functions. This technique requires power return measured as a function of incidence angle at a small angular interval and is not readily applicable when this is not true.

Stiles et al. [2] described an empirical procedure to estimate the error, when the narrow-beam approximation is used for small incidence angles, by initially hypothesizing the theoretical $\sigma^o(\theta)$ curve.

Moore [3] suggested an iteration scheme to estimate the scattering coefficient by using the constant-σ^o approximation to get a first estimate of the σ^o variation function and then continuously refining that function to get better estimate of σ^o. This method is used in this report, and the integral is evaluated numerically using the real system parameters of the HELOSCAT III FM-CW scatterometer and a data set taken from sea ice.

2.0 NARROW-BEAM APPROXIMATION

Consider the radar equation

$$P_r = \iint\limits_{\substack{\text{area}\\ \text{illuminated}}} \frac{P_t G^2 \lambda^2 \sigma^o}{(4\pi)^3 R^4}\, dA \tag{1}$$

where identical transmit- and receive-antenna gains are used.

The first problem here is how to define the illuminated area. Actually the antennae have lobes everywhere, so the area has to be from $-\infty$ to $+\infty$ in the plane of illumination. Usual practice for this is to assume that the antenna beam is confined in a finite cone within which the maximum value of gain applies, with zero gain elsewhere. The effective beamwidth, β_{eff}, is defined in such a way that the pattern solid angles for the two-way real antenna beam and the cone-shaped beam are the same (see Figure 1). Therefore,

$$\iint\limits_{4\pi} G^2(\theta,\phi)\, d\Omega = \int_0^{2\pi}\int_0^{\pi} G^2(\theta,\phi) \sin\theta d\theta d\phi$$

$$\triangleq \int_0^{2\pi}\int_0^{(\frac{\beta_{eff}}{2})} G_0^{\,2} \sin\theta d\theta d\phi \tag{2}$$

where $G_0^{\,2}$ is the maximum gain for the two-way pattern. This is not a simple task unless $G^2(\theta,\phi)$ is completely known. If the gain function is assumed to be a Gaussian and if it is ϕ-independent, then

$$2\pi \int_0^{\pi} G_0^{\,2}\, e^{-2.773\theta^2/\beta_{3dB}^2} \sin\theta d\theta \triangleq 2\pi \int_0^{\frac{\beta_{eff}}{2}} G_0^{\,2} \sin\theta d\theta \tag{3}$$

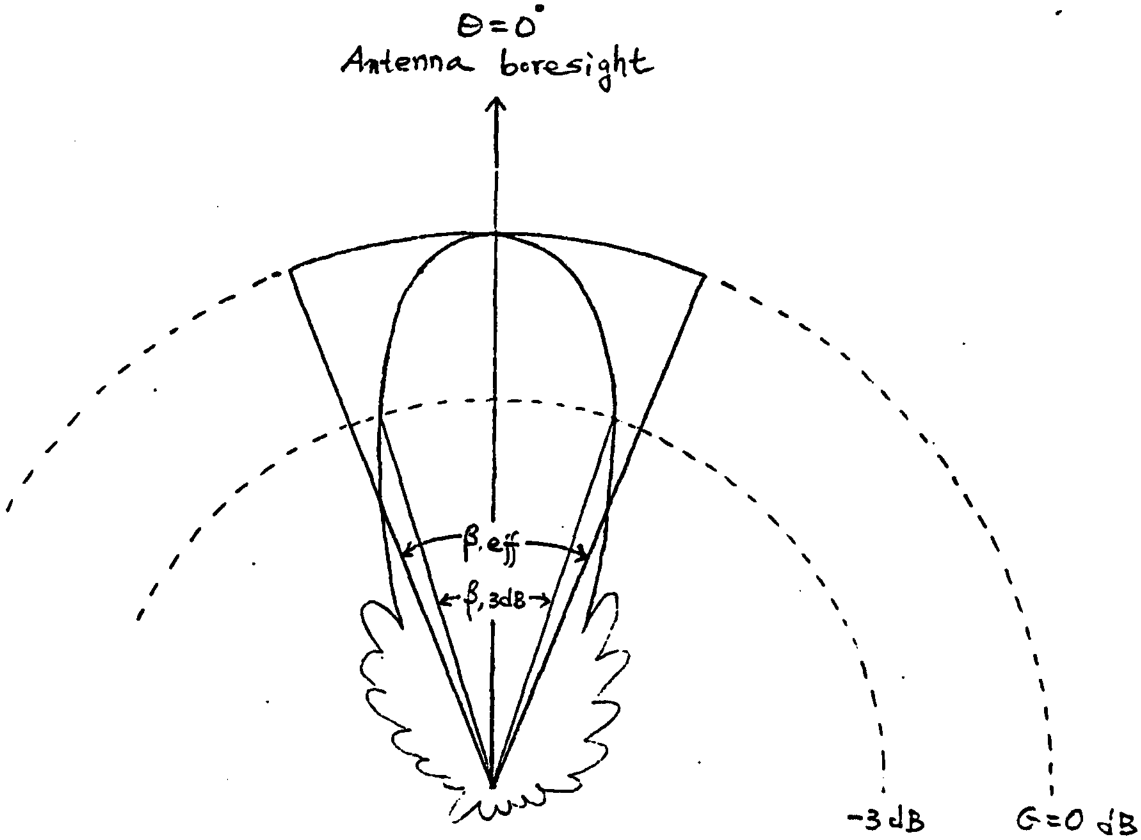

Figure 1: Effective Beamwidth and Half-Power Beamwidth

which reduces to

$$\beta_{eff} = 2 \cos^{-1}\left(1 - \int_0^{\pi} e^{-2.773\theta^2/\beta_{3dB}^2} \sin\theta d\phi\right) \tag{4}$$

This equation can be solved numerically and for the β_{3dB} of up to 15°, the following relation can be used.

$$\beta_{eff} = 1.2\ \beta_{3dB} \tag{5}$$

Antenna beamwidths are usually measured in azimuth and elevation planes. When the measured patterns are considerably different from each other, separate effective beamwidths are defined.

$$\beta_{a,eff} = 1.2\ \beta_{a,3dB} \tag{6}$$

$$\beta_{e,eff} = 1.2\ \beta_{e,3dB} \tag{7}$$

The illuminated area is then the ground projection (Figure 2) of the conical beam and the area of the ellipse is approximately

$$A_i = \frac{\pi h^2}{2\cos\theta_p}\left[\tan\left(\theta_p + \frac{\beta_{e,eff}}{2}\right) - \tan\left(\theta_p - \frac{\beta_{e,eff}}{2}\right)\right] \tan\left(\frac{\beta_{a,eff}}{2}\right) \tag{8}$$

If the beamwidths are narrow enough, R^4 inside the integral in Eq. (1) can be treated to be constant; and if σ^o varies slowly inside the illuminated area given by Eq. (8), one can remove the σ^o from the integral and solve for σ^o.

$$\sigma^o = \frac{(4\pi)^3}{\lambda^2} \left(\frac{P_r}{P_t}\right) \left(\frac{R_o^4}{G_o^2}\right) \frac{1}{A_i} \tag{9}$$

This is the normal practice. The variations in transmitted power, P_t and G_o^2 are found using a delay line calibration scheme and standard point-target measurements, respectively. However, these approximations of constant-gain,

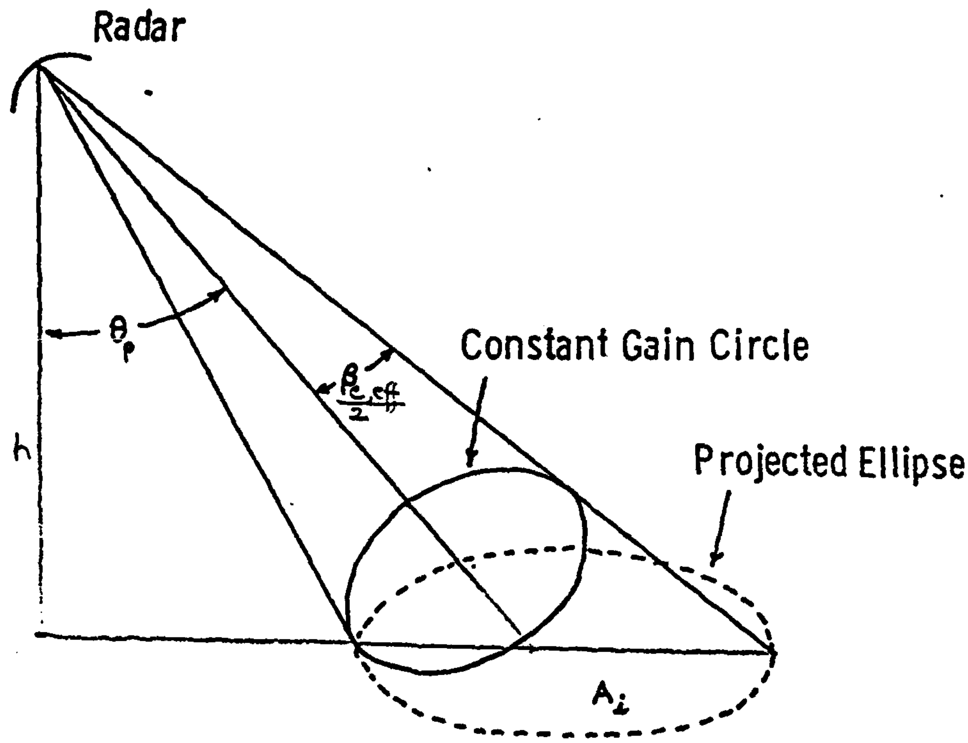

Figure 2: Effective Illuminated Area

finite-beam, constant-range, constant-σ^o inside the integral may not be true for a wide-beam, an antenna pattern with high sidelobes, near-nadir or near grazing angles of incidence.

3.0 EFFECT OF ANTENNA GAIN AND RANGE VARIATION INSIDE THE INTEGRAL

As a first attempt, the effects of a more realistic antenna gain and R^4 inside the integral are evaluated. The effect of σ^o variation across the illuminated area will be treated in Section 4.0.

If a full set of antenna patterns in all directions is available, these numerical values can be entered as a gain function. But this is not usually the situation and some kind of functional approximation of the antenna gain is inevitable.

3.1 Gaussian Antenna Gain

If the antenna gain can be represented as a Gaussian, then

$$G^2 = G^2(\theta_a,\phi_a) = G_o^{\,2} \exp(-2.773(\theta_a-\theta_p)^2/\beta_{e,3dB}^{\,2})\exp(-2.773\phi_a^{\,2}/\beta_{a,3dB}^{\,2}) \quad (10)$$

where:

θ_p = antenna boresight pointing angle off vertical (in x-z plane)

θ_a = angle off vertical in elevation plane for observation direction

ϕ_a = angle off x-z plane for observation direction

$\beta_{e,3dB}$ = two-way 3 dB beamwidth in elevation plane

$\beta_{a,3dB}$ = two-way 3 dB beamwidth in azimuth plane

For point P(x,y) on the ground (see Figure 3),

$$\theta_a = \tan^{-1}(x/h) \quad (11)$$

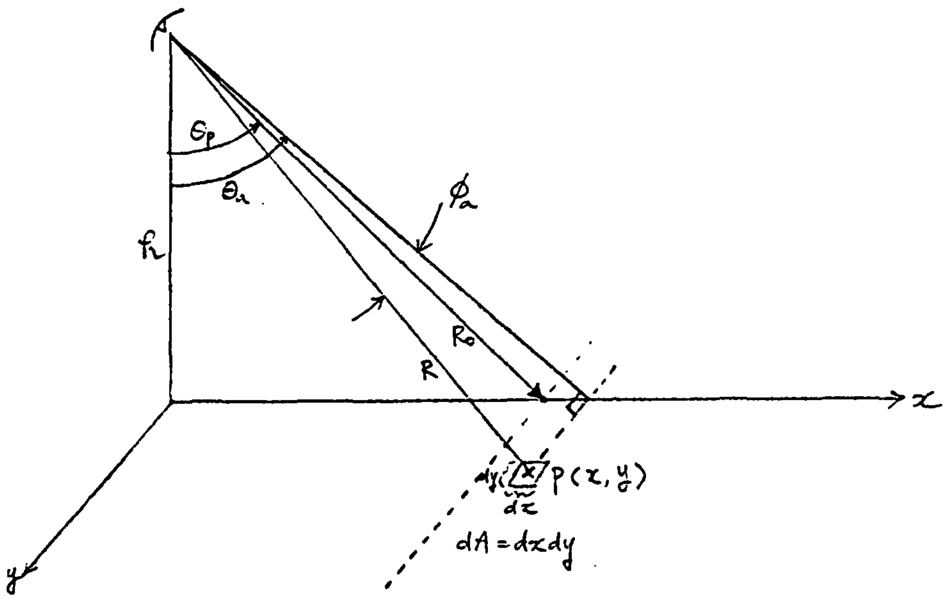

Figure 3: Coordinate System for the Antenna Gain Functions

$$\phi_a = \tan^{-1}(y/\sqrt{x^2+h^2}) \tag{12}$$

This coordinate system is not a spherical coordinate system, but this is used because the antenna patterns are available in these azimuth and elevation planes.

3.2 $(\sin x/x)^4$ Antenna Gain

One-way antenna gain can also be represented by $(\sin x/x)^2$. In terms of one-way 3 dB beamwidth, β'_{3dB},

$$G(\theta) = G_o\left[\frac{\sin(2.7831\theta/\beta'_{3dB})}{2.7831\theta/\beta'_{3dB}}\right]^2 \tag{13}$$

Therefore, in the coordinate system shown in Figure 3, two-way antenna gain can be represented as

$$G^2(\theta_a,\phi_a) = G_o^{\,2}\left(\frac{\sin\chi_\theta}{\chi_\theta}\right)^4\left(\frac{\sin\chi_\phi}{\chi_\phi}\right)^4 \tag{14}$$

where:

$\chi_\theta = 2.7831\ (\theta_a - \theta_p)/\ _{a,3dB}$

$\chi_\phi = 2.7831\ \phi_a/\beta'_{a,3dB}$

$\beta'_{e,3dB}$ = one-way 3 dB beamwidth in elevation plane

$\beta'_{a,3dB}$ = one-way 3 dB beamwidth in azimuth plane

θ_a, ϕ_a = given in Eqs. (11) and (12)

This two-way gain function given by Eq. (14) has many sidelobes and nulls, with the highest sidelobes 26.5 dB down from mainlobe (see Figure 4). The location of the first sidelobes will be about 1.6 β'_{3dB} away from the center of the mainlobe.

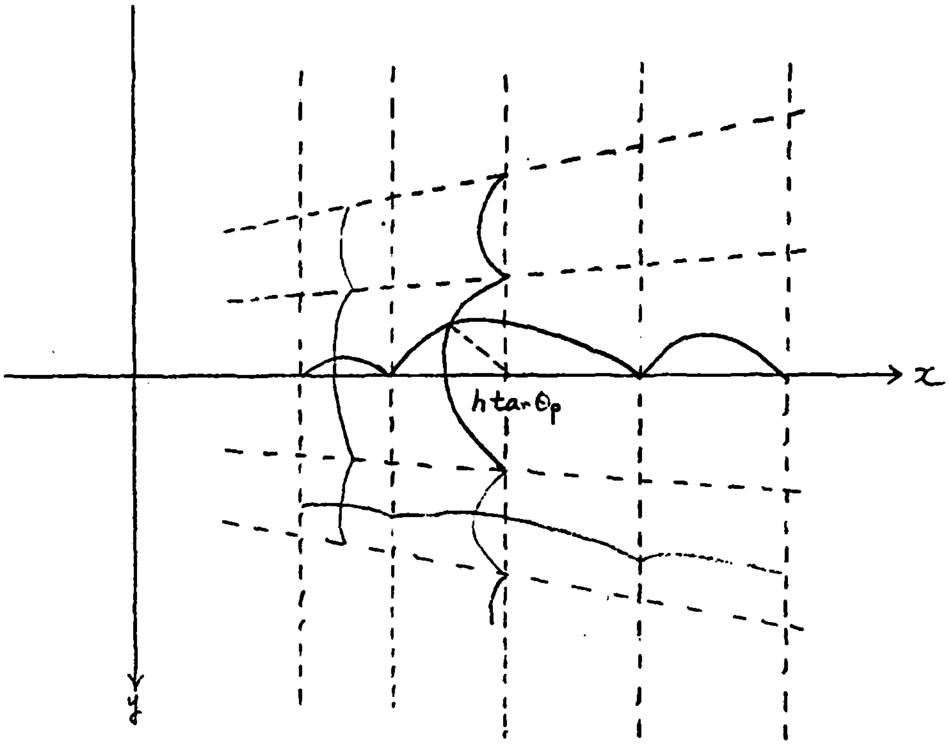

Figure 4: $\left(\frac{\sin\chi}{\chi}\right)^4$ Antenna Gain Projected on the Ground

3.3 Range Variation and the Effect of the Bandpass Filter

The range R from the antenna to the infinitesimal area element dA on the ground is

$$R = \sqrt{\chi^2+y^2+h^2} \qquad (15)$$

In the actual HELOSCAT III system (an FM radar) the range to the target is limited by the bandpass filter in the final stage. Each area element on the ground has different range to the antenna and thereby gives a different IF frequency. Therefore a bandpass filter with finite bandwidth would limit the range from which the radar receives the scattered field. In the narrow-beam approximation, the bandpass filter plays a role only when the effective beamwidth is very wide and the incidence angle is large enough. In such a case, the filter cuts the area of the ellipse from which σ^o is calculated.

The transfer function of the bandpass filter shown in Figure 5 can be represented as a function of range as follows.

$$
\begin{aligned}
f(R) &= 1, && \text{when } R_o(1 - B'/2f_o) \le R \le R_o(1 + B'/2f_o) \\
&= 10^{\frac{145}{4}\left(\frac{R}{R_o} - (1-\beta'/2f_o)\right)} && \text{when } R \le R_o(1 - B'/2f_o) \\
&= 10^{-30\left(\frac{R}{R_o} - (1+\beta'/2f_o)\right)} && \text{when } R \ge R_o(1 + B'/2f_o) \qquad (16)
\end{aligned}
$$

where:

R_o is the distance to the center of the illuminated area

B' is the bandwidth of the filter (Figure 5),

f_o is the center frequency of the filter.

Theoretically in a FM-CW radar, the range R to the target and the IF frequency output (F_{if}) of the mixer have the following relationship.

$$F_{if} = \frac{4R\Delta F\ F_m}{c} \tag{17}$$

where:

ΔF = RF frequency sweep width

F_m = modulating frequency

c = velocity of light

A sample of the transmitted waveform long before it goes out of the antenna is usually used as the local oscillator signal to be mixed with the received signal. Hence the actual IF frequency is different from what one expects from Eq. (17), especially when long antenna cables are used. The discrepancy in range should be obtained experimentally as a system characteristic. When this additional range, R_c, is not considered to be negligible, Eq. (17) and the bandpass filter function given by Eq. (16) should be modified accordingly (see Figure 5).

$$F_{if} = \frac{4(R+R_c)\Delta F\ F_m}{c} \tag{18}$$

$$\begin{aligned} f(R) &= 1, \quad \text{when } R_o(1-B'/2f_o)-(B'/2f_o)R_c \le R \le (1+B'/2_{fo})R_o+(B'/2_{fo})R_c \\ &= 10^{\frac{145}{4}\left(\frac{R+R_c}{R_o+R_c} - (1-B'/2_{fo})\right)}, \text{ when } R \le (1-B'/2_{fo})R_o - (B'/2f_o)R_c \\ &= 10^{-30\left(\frac{R+R_c}{R_o+R_c} - (1+B'/2f_o)\right)}, \text{ when } R \ge (1+B'/2f_o)R_o + (B'/2_{fo})R_c \end{aligned} \tag{19}$$

Examining Eq. (19), one can see that having some additional range, R_c, has effectively widened the bandwidth of the filter.

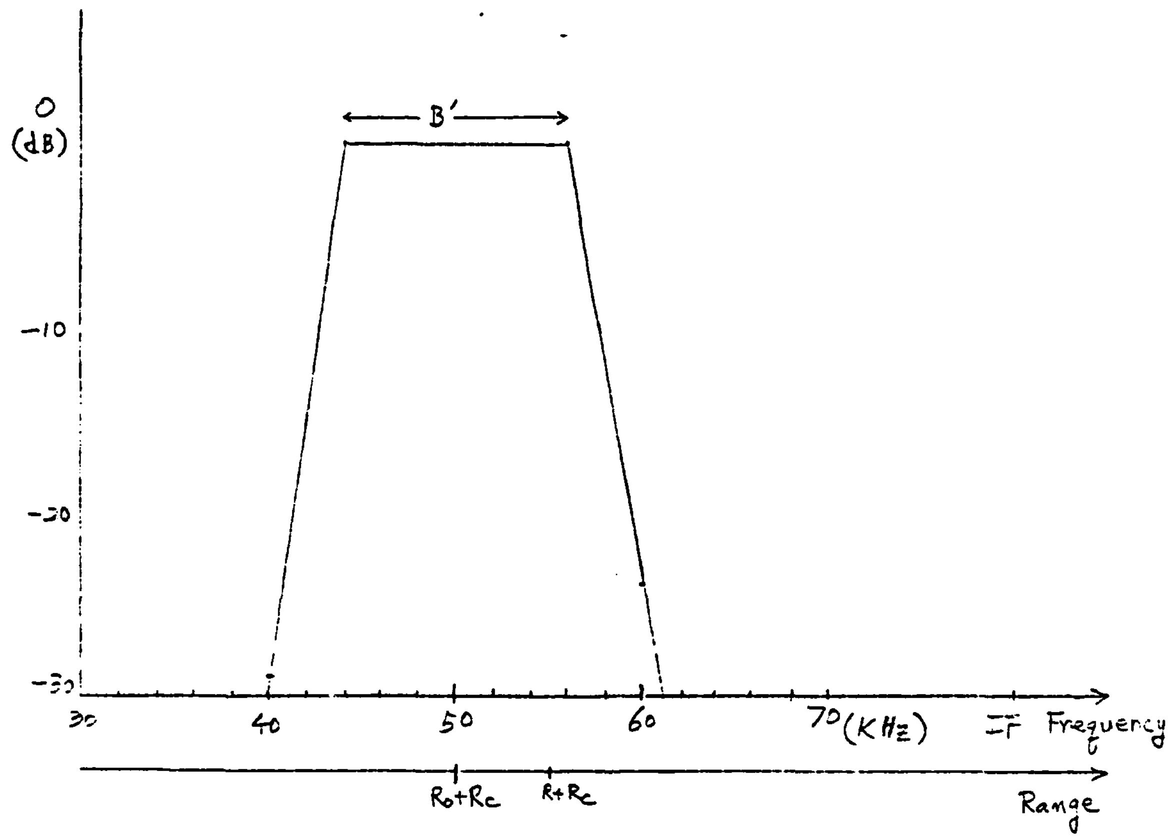

Figure 5: Bandpass Filter Function

3.4 Integration Limits

In this section, σ^o will be assumed to be constant across the integration limits and evaluated using the following equation.

$$\sigma^o = \frac{(4\pi)^3}{\lambda^2} \left(\frac{P_r}{P_t}\right) \frac{1}{\iint \frac{G^2(\theta_a,\phi_a) f(R)}{R^4} dA} \tag{20}$$

In view of the narrow beam approximation given by Eq. (9), one can rewrite this equation as

$$\sigma^o = \frac{(4\pi)^3}{\lambda^2} \left(\frac{P_r}{P_t}\right) \left(\frac{R_o^4}{G_o^2}\right) \frac{1}{\iint \frac{g^2(\theta_a,\phi_a) f(R)}{(R/R_o)^4} dA} \tag{21}$$

where $G^2(\theta_a,\phi_a) = G_o^2 g^2(\theta_a,\phi_a)$.

The integral can be treated as a weighted area [4] compared to the illuminated area described in Section 2.0.

$$A_w \triangleq \iint \frac{g^2(\theta_a,\phi_a) f(R)}{(R/R_o)^4} dA \tag{22}$$

The integration is done using a rectangular coordinate system. Therefore the infinitesimal area $dA = dx\, dy$ and the integration limits should be the whole ground plane and the filter function will effectively set the limit. To simplify the numerical integration, the integration limits were set to be the points where the antenna gain function reduces to small enough numbers (about -30 dB) for the Gaussian gain, and for the $(\sin x/x)^4$ gain function the limits were up to the first sidelobes. In this case the integral is also evaluated for the mainlobe only to see the effect of sidelobes. Figure 6 shows the integration limits (square area marked -30 dB on four sides)

together with the elliptical (shaded) illuminated area normally used for the narrow-beam approximation. Also, the equal-range circles which effectively limit the area by the filter function can be seen.

The weighted area now becomes

$$A_w = \int_{y-}^{y+}\int_{x}^{x+} \frac{g^2[\theta_a(x,y),\phi_a(x,y)]f[R(x,y)]}{[R(x,y)/R_o]^4}\, dx\, dy \qquad (23)$$

where g^2, R and f are given by Eqs. (10-12), (14), (15), (19) and

$$x\pm = h\tan(\theta_p \pm 1.6\ \beta_{e,3dB}) \text{ for Gaussian gain}$$

$$= h\tan(\theta_p \pm 3.2\ \beta_{e,3dB}) \text{ for } (\sin x/x)^4\text{, up to first sidelobes}$$

$$= h\tan(\theta_p \pm 1.6\ \beta_{e,3dB}) \text{ for } (\sin x/x)^4\text{, mainlobe only}$$

and

$$y\pm = R_o\tan(\pm 1.6\ \beta_{a,3dB}) \text{ for Gaussian gain}$$

$$= R_o\tan(\pm 3.2\ \beta_{a,3dB}) \text{ for } (\sin x/x)^4\text{, up to first sidelobes}$$

$$= R_o\tan(\pm 1.6\ \beta_{a,3dB}) \text{ for } (\sin x/x)^4\text{, mainlobe only}$$

and

$$R_o = h/\cos\theta_p$$

3.5 Results

In this section, the possible error when using constant range and constant gain across the effective beamwidth is defined as follows.

$$\text{ERROR1} = A_i/A_w \qquad (24)$$

where:

A_w is the weighted area given by Eq. (23) and

A_i is the area of the ellipse given by Eq. (8)

then

$$\text{ERROR1 (dB)} = 10 \log(A_i/A_w) \tag{25}$$

and this ERROR1 (dB) should be added to the σ^0(dB) calculated using a narrow-beam approximation.

The numerical integration was done using Gauss' formula [5] and a total of 1600 (40x40) points inside the integration limit shown in Figure 6. Because the number of points evaluated was fixed for all the incidence angles between 0° to 70° while the actual integration limits in x- and y-coordinate grew tremendously from 0° to 70°, the accuracy of the numerical integration tends to get worse at large incidence angles.

Table 1 is a summary of ERROR1 for various antenna beamwidths and antenna gain functions. As can be seen in the table, the constant-range and constant-gain across the effective beamwidths are not bad approximations to use up to the pointing angle of 60°, except when the two-way beamwidths are as large as 9°, if the variations of σ^0 inside the integral can be neglected as assumed here.

4.0 EFFECT OF σ^0 VARIATION INSIDE THE INTEGRAL

Up to now, σ^0 was assumed to be constant within the limits of integration, so σ^0 was outside the integral. Actually, σ^0 itself is a function of the local incidence angle and therefore a function of x- and y-coordinates of the point evaluated (see Figure 7).

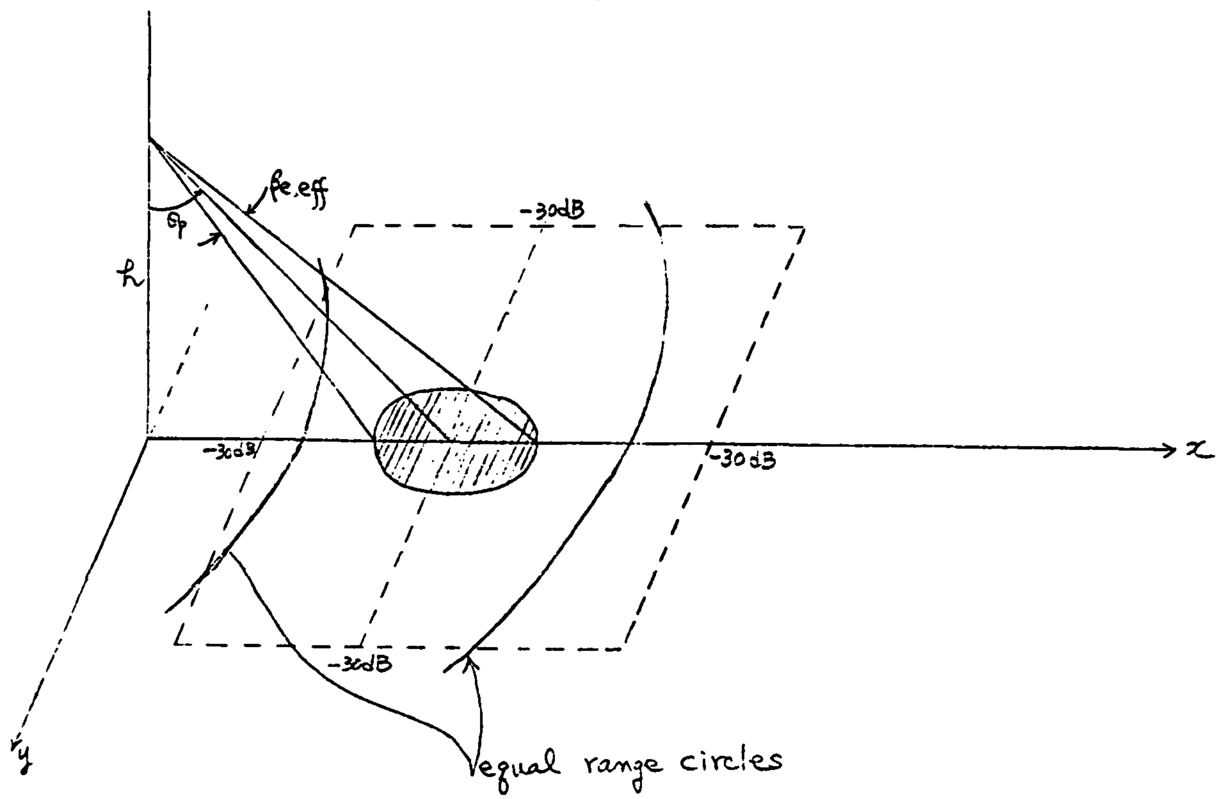

Figure 6: Integration Limits

TABLE 1

ERROR1

	Pointing Angle θ_p(°)	Two-Way 3dB Beamwidths $\beta_e = 9.0°$ $\beta_a = 9.0°$	$\beta_e = 6.3°$ $\beta_a = 5.7°$	$\beta_e = 5.0°$ $\beta_a = 5.0°$	$\beta_e = 2.4°$ $\beta_a = 2.0°$
Gaussian Gain	0	0.055(dB)	0.025(dB)	0.017(dB)	0.004(dB)
	10	0.057	0.025	0.018	0.004
	20	0.060	0.027	0.019	0.004
	30	0.069	0.031	0.021	0.005
	40	0.095	0.038	0.026	0.006
	50	0.208	0.063	0.035	0.008
	60	0.591*	0.222	0.094	0.012
	70	0.366*	0.616	0.588	0.033
$\left(\frac{\sin x}{x}\right)^4$ up to first sidelobes	0	0.059	0.029	0.022	0.008
	10	0.060	0.029	0.022	0.008
	20	0.067	0.031	0.023	0.008
	30	0.076	0.037	0.026	0.009
	40	0.094	0.046	0.033	0.010
	50	0.228	0.070	0.044	0.012
	60	0.583*	0.206	0.091	0.021
	70	0.484*	0.484	0.528	0.042
$\left(\frac{\sin x}{x}\right)^4$ mainlobe only	0	0.079	0.049	0.043	0.030
	10	0.080	0.050	0.043	0.030
	20	0.084	0.052	0.044	0.030
	30	0.092	0.056	0.047	0.030
	40	0.111	0.063	0.051	0.031
	50	0.208	0.080	0.060	0.033
	60	0.561*	0.213	0.102	0.038
	70	0.340*	0.582	0.557	0.052

* These values may not be very accurate due to the large spread between points actually evaluated.

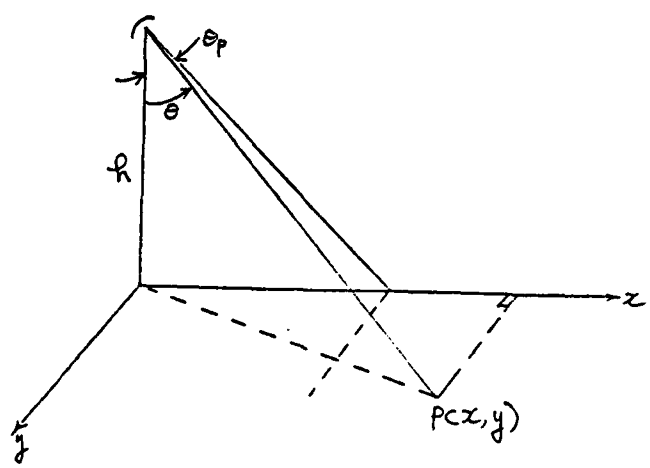

Figure 7: Local Incidence Angle

$$\sigma^o = \sigma^o(\theta) = \sigma^o(x,y)$$

where θ is the local incidence angle and

$$\theta = \tan^{-1} \frac{\sqrt{x2+ y2}}{h} \tag{26}$$

One way of evaluating σ^o suggested in [3] is to apply Eq. (21) to a set of measurements to get a rough estimate of σ^o variation with incidence angle, then use this function in the integral (Eq. (1)), and continuously refine the σ^o variation function to get better estimate of σ^o.

Variation of σ^o with incidence angle will take many forms depending on the kind of target. In many cases, including sea ice, it fits quite well with the exponential function used here. From Eqs. (1) and (21),

$$P_r = \frac{\lambda^2}{(4\pi)^3} P_t \frac{G_o^2}{R_o^4} \iint \frac{g^2(\theta_a,\phi_a) f(R) \sigma^o(\theta)}{(R/R_o)^4} dA \tag{27}$$

and $\sigma^o(\theta)$ for some particular angle of incidence can be evaluated as,

$$\sigma_2^o(\theta_p) = \frac{(4\pi)^3}{\lambda^2} \left(\frac{P_r}{P_t}\right)\left(\frac{R_o^4}{G_o^2}\right) \frac{1}{\iint \frac{g^2 \; f \; [\sigma_1(\theta)/\sigma_1(\theta_p)]}{(R/R_o)^4} dA} \tag{28}$$

where $\sigma_1^o(\theta)$ is the function one gets by applying Eq. (21) to a set of measurements. The next iteration becomes

$$\sigma_3^o(\theta_p) = \frac{(4\pi)^3}{\lambda^2} \left(\frac{P_r}{P_t}\right)\left(\frac{R_o^4}{G_o^2}\right) \frac{1}{\iint \frac{g^2 \; f \; [\sigma_2(\theta)/\sigma_2(\theta_p)]}{(R/R_o)^4} dA} \tag{29}$$

and so on.

If we assume the form $\sigma^o(\theta) = ae^{-b\theta}$, then

$$\sigma_1^o(\theta)/\sigma_1^o(\theta_p) = e^{-b_1(\theta-\theta_p)} \text{ and } \sigma_2^o(\theta)/\sigma_2^o(\theta_p) = e^{-b_2(\theta-\theta_p)}$$

and σ^o variation function around the pointing angle, θ_p, can be defined as

$$S_n(\theta,\theta_p) \triangleq \frac{\sigma_n^o(\theta)}{\sigma_n^o(\theta_p)} = e^{-b_n(\theta-\theta_p)} \tag{30}$$

and the n-th refined σ^o becomes

$$\sigma_n^o(\theta_p) = \frac{(4\pi)^3}{\lambda^2}\left(\frac{P_r}{P_t}\right)\left(\frac{R_o^4}{G_o^2}\right)\frac{1}{\iint \frac{g^2 f S_{n-1}(\theta,\theta_p)}{(R/R_o)^4} dA} \tag{31}$$

Hence the integral can be treated as another weighted area similar to that of Eq. (23).

$$A_{ws} \triangleq \iint \frac{g^2 f S_{n-1}}{(R/R_o)^4} dA \tag{32}$$

The possible error when the σ^o variation function is neglected is defined as

$$\text{ERROR2(dB)} = 10 \log(A_w/A_{ws}) \tag{33}$$

and numerically evaluated (Table 2) for several antenna gain functions, beam-widths and σ^o variation across the integration limits (assuming that the variation is known in advance). In Table 2, the functions of $e^{-\theta/12.9°}$ and $e^{-\theta/9.4°}$ were derived from linear regression (in dB scale) of the data points obtained from sea ice using the narrow beam approximation and the steep variation function of $e^{-\theta/5°}$ is appropriate to some open water.

In the next section, the iteration will be tried. The error caused by not considering steep σ^o variation can be as large as 3.6 dB for normal

TABLE 2

ERROR2

	Pointing Angle θ_p(°)	σ^o-variation and Two-way 3 dB Beamwidths			
		$\sigma^o = e^{-\theta/5°}$ $\beta_e = 9.0°$ $\beta_a = 9.0°$	$\sigma^o = e^{-\theta/12.9°}$ $\beta_e = 6.3°$ $\beta_a = 5.7°$	$\sigma^o = 3^{-\theta/5°}$ $\beta_e = 5.0°$ $\beta_a = 5.0°$	$\sigma^o = e^{-\theta/9.4°}$ $\beta_e = 2.4°$ $\beta_a = 2.0°$
Gaussian Gain	0	3.647(dB)	1.040(dB)	2.140(dB)	0.532(dB)
	10	-0.484	0.004	-0.193	-0.010
	20	-0.994	-0.060	-0.314	-0.020
	30	-1.170	-0.087	-0.365	-0.025
	40	-1.301	-0.108	-0.401	-0.029
	50	-1.456	-0.133	-0.438	-0.033
	60	-1.386	-0.190	-0.505	-0.038
	70	-0.398	-0.184	-0.548	-0.050
$(\frac{\sin x}{x})^4$ up to first sidelobes	0	3.613	1.026	2.114	0.526
	10	-0.450	0.008	-0.184	-0.010
	20	-1.015	-0.061	-0.326	-0.020
	30	-1.330	-0.089	-0.377	-0.025
	40	-1.405	-0.111	-0.417	-0.029
	50	-1.359	-0.122	-0.437	-0.033
	60	-1.366	-0.176	-0.458	-0.040
	70	-0.633	-0.205	-0.555	-0.044
$(\frac{\sin x}{x})^4$ mainlobe only	0	3.574	1.012	2.090	0.517
	10	-0.466	0.003	-0.177	-0.009
	20	-0.904	-0.055	-0.289	-0.019
	30	-1.063	-0.080	-0.336	-0.023
	40	-1.177	-0.099	-0.369	-0.027
	50	-1.339	-0.120	-0.402	-0.030
	60	-1.388	-0.177	-0.459	-0.035
	70	-1.010	-0.186	-0.535	-0.044

incidence and in the order of 1 dB for other incidence angles when the beamwidths are 9°. When the beamwidths are 5°, the error is 2.1 dB for normal incidence and less than 0.5 dB for other incidence angles. When the σ^o variation is not so rapid ($e^{-\theta/12.9°}$ or $e^{-\theta/9.4°}$), the error reduces to 1 dB or less for normal incidence and becomes very small for other incidence angles. Recall from Section 3.5 that the numerical integration may have some error for large incidence angles. Table 2 also shows that the $(\sin x/x)^4$ gain function has such a low sidelobe level (-26.5 dB) that integrating up to first sidelobes did not differ significantly from integrating only the mainlobe. Furthermore, the $(\sin x/x)^4$ antenna gain function and the Gaussian gain function did not show very much difference.

4.1 Iteration

As explained in the previous section, iteration is tried with real data and some arbitrary data which fits well with the exponential function. Because the real data points are from 10° to 70°, only these points are evaluated to get an exponential fit and then iterated several times. Table 3 is a summary of the result. The first row shows the σ^o calculated using the narrow beam approximation (σ^o_{nb}) and the second row is the modified σ^o computed using gain function and range variations (σ^o_{nb} + ERROR1). These points are regressed to get an exponential fit, and this exponential function is included in the integration to give the third row (σ^o_{nb} + ERROR1 + ERROR2). From these points we get new slope and therefore new ERROR2, and for the fourth row, $\sigma^o = \sigma^o_{nb}$ + ERROR1 + ERROR2, and so on.

TABLE 3
σ^0 DATA, ORIGINAL AND UP TO 4th REFINED

1. 13.6 GHz Sea Ice Data ($\sigma^0 \approx e^{-\theta/9.4°}$)
$\beta_e = 2.4°$ $\beta_a = 2.0°$

	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE	CORRELATION
Gaussian Gain		2.70	2.60	-1.70	-6.80	-10.00	-14.90	-26.00	-0.4621	-0.9643
		2.71	2.61	-1.69	-6.79	-9.99	-14.88	-25.96	-0.4617	-0.9644
		2.70	2.59	-1.72	-6.82	-10.02	-14.92	-26.00	-0.4622	-0.9645
		2.70	2.59	-1.72	-6.82	-10.02	-14.92	-26.00	-0.4622	-0.9645
		2.70	2.59	-1.72	-6.82	-10.02	-14.92	-26.00	-0.4622	-0.9645
$\left(\frac{\sin x}{x}\right)^4$, up to first sidelobes	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE	CORRELATION
		2.70	2.60	-1.70	-6.80	-10.00	-14.90	-26.00	-0.4621	-0.9643
		2.70	2.60	-1.70	-6.79	-9.99	-14.89	-25.97	-0.4618	-0.9644
		2.69	2.58	-1.72	-6.82	-10.03	-14.93	-26.02	-0.4623	-0.9645
		2.69	2.58	-1.72	-6.82	-10.03	-14.93	-26.02	-0.4623	-0.9645
		2.69	2.58	-1.72	-6.82	-10.03	-14.93	-26.02	-0.4623	-0.9645

2. 4.8 GHz Sea Ice Data ($\sigma^0 \approx e^{-\theta/12.9°}$)
$\beta_e = 6.3°$ $\beta_a = 5.7°$

	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE	CORRELATION
Gaussian Gain		-2.70	-4.90	-10.50	-12.30	-14.80	-18.00	-24.00	-0.3371	-0.9886
		-2.67	-4.87	-10.47	-12.26	-14.74	-17.78	-23.38	-0.3293	-0.9893
		-2.67	-4.93	-10.55	-12.36	-14.87	-17.96	-23.56	-0.3324	-0.9901
		-2.67	-4.93	-10.55	-12.37	-14.87	-17.97	-23.56	-0.3324	-0.9901
		-2.67	-4.93	-10.55	-12.37	-14.87	-17.97	-23.56	-0.3324	-0.9901
$\left(\frac{\sin x}{x}\right)^4$, up to first sidelobes	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE	CORRELATION
		-2.70	-4.90	-10.50	-12.30	-14.80	-18.00	-24.00	-0.3371	-0.9886
		-2.67	-4.87	-10.46	-12.25	-14.73	-17.79	-23.52	-0.3309	-0.9894
		-2.66	-4.93	-10.55	-12.36	-14.85	-17.97	-23.72	-0.3341	-0.9896
		-2.66	-4.93	-10.55	-12.36	-14.85	-17.97	-23.72	-0.3341	-0.9896
		-2.66	-4.93	-10.55	-12.36	-14.85	-17.97	-23.72	-0.3341	-0.9896

3. Arbitrary Data ($\sigma^0 \approx e^{-\theta/5°}$ $\beta_e = \beta_a = 9°$)

	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE	CORRELATION
Gaussian Gain		-8.70	-17.00	-26.60	-33.00	-42.00	-53.00	-60.00	-0.8618	-0.9987
		-8.64	-16.94	-26.53	-32.91	-41.79	-52.41	-59.63	-0.8542	-0.9989
		-9.11	-17.90	-27.67	-34.17	-43.22	-53.78	-60.62	-0.8636	-0.9989
		-9.13	-17.93	-27.69	-34.20	-43.25	-53.80	-60.63	-0.8636	-0.9989
		-9.13	-17.93	-27.69	-34.20	-43.25	-53.80	-60.63	-0.8636	-0.9989
$\left(\frac{\sin x}{x}\right)^4$ up to first sidelobes	ANGLE	10°	20°	30°	40°	50°	60°	70°	SLOPE	CORRELATION
		-8.70	-17.00	-26.60	-33.00	-42.00	-53.00	-60.00	-0.8618	-0.9987
		-8.64	-16.93	-26.52	-32.91	-41.77	-52.42	-59.52	-0.8530	-0.9988
		-9.07	-17.92	-27.81	-34.27	-43.10	-53.76	-60.14	-0.8577	-0.9987
		-9.08	-17.93	-27.83	-34.29	-43.11	-53.77	-60.14	-0.8577	-0.9987
		-9.08	-17.93	-27.83	-34.29	-43.11	-53.77	-60.14	-0.8577	-0.9987

For all the cases in Table 3, the data seems to converge after three iterations, so it did not require further iteration. Also from Table 1 and Table 2, it can be seen that the ERROR1 and ERROR2 tend to cancel out each other to make σ^o_{nb} a tolerable estimation at least for the incidence angles of 10° to 70° and when the beamwidths are not very wide.

5.0 SUMMARY

Three kinds of so-called illuminated areas are treated in this report:

(1) the area of an ellipse with constant gain and constant range to the antenna for all the cells within the ellipse, A_i, (Eq. (8)).

(2) the area weighted with antenna gain and range, A_w (Eq. (23)).

(3) the area weighted with antenna gain and range and also σ^o variation, A_{ws}, (Eq. (32)).

The possible error when using A_i instead of A_w was defined as ERROR1 and summarized in Table 1. When the σ^o variation with incidence angle is small, the result shows that the constant range and constant gain approximation gives a result within 0.6 dB or less.

Next, σ^o variation is included and the possible error of using A_w instead of A_{ws} was defined as ERROR2 and listed in Table 2. For normal incidence, one can have up to 3.6 dB error when the beamwidth is 9° for rapidly varying ($e^{-\theta/5°}$)σ^o. The big error at normal incidence is always present and one must consider this problem when σ^o for normal incidence is evaluated. The error will be bigger than the values obtained here if the real σ^o variation near vertical is steeper than the exponential function, which is often true for very smooth targets.

The more realistic σ^{o} values corrected with these estimated errors were further refined using iterative scheme. Actually, however, the iteration did not show any further improvement because it converged almost instantly for the exponential σ^{o} variation function assumed in this report. This may not be true for other kinds of σ^{o} variation functions and can be further investigated using the technique described in this report.

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APPENDIX
PROGRAM LISTING

```
C       THIS PROGRAM IS TO EVALUATE THE POSSIBLE ERROR WHEN
C       USING THE NARROW BEAM APPROXIMATION IN CALCULATING
C       THE SCATTERING COEFFICIENT.
C       ANTENNA GAIN FUNCTIONS,GAUSSIAN AND (SIN(X)/X)**4,
C       AND REAL RANGE AND TRAPEZOIDAL BAND PASS FILTER FUNCTION
C       AND THE EXPONENTIAL SIGMA VARIATION FUNCTION IS EVALUATED
C       IN THE INTEGRAL USING GAUSS' FORMULA. 40 POINT BY 40 POINT
C       TOTAL 1600 POINTS ARE EVALUATED IN THE INTEGRATION LIMIT.
C
C        FUNCTIONS REFERENCED
C          GAIN1=GAUSSIAN ANTENNA GAIN
C          GAIN2=(SIN(X)/X))**4 GAIN
C          BPF1=BANDPASS FILTER FUNCTION
C          ASIGMA=EXPONENTIAL SIGMA VARIATION FUNCTION
C          REGRES=LINEAR REGRESSION ROUTINE
C          AVG=AVERAGING ROUTINE
C          AREA=EFFECTIVE AREA CALCULATING ROUTINE FOR THE
C               NARROW BEAM APPROXIMATION CASE
C        VARIABLES USED
C          BE=TWO WAY 3 DB ELEVATION BEAMWIDTHS
C          BA=TWO WAY 3 DB AZIMUTH BEAMWIDTHS
C          ANGLE=POINTING ANGLE
C          THEO=ANGLE IN RADIANS
C          W(40),X(40)=GAUSS' CONSTANTS FOR INTEGRATION
C          RO=RANGE TO THE BEAM CENTER
C          UPX,UPY=UPPER LIMITS IN X AND Y COORDINATES
C          LOWX,LOWY=LOWER LIMITS
C          H=ANTENNA HEIGHT
C          ERROR1=ERROR FOR NOT COUNTING GAIN AND RANGE VARIATION
C          ERROR2=ERROR FOR NOT COUNTING SIGMA
C          EAREA=EFFECTIVE AREA OF THE ELLIPSE
C          WAREA=GAIN AND RANGE WEIGHTED AREA
C          RAREA=SIGMA,GAIN,RANGE WEIGHTED ARES
C          AL=CONST TO  SET THE LIMIT OF INTEGRATION
C
        COMMON BE,BA
        REAL BE,BA
        REAL ANGLE,THEO,PI,W(40),X(40),RO,UPX,LOWX,UPY,LOWY
        REAL HALFX,HALFY,AVGX,AVGY,ISUM,JSUM,XX,YY,H,G,F
        REAL ISUM1,JSUM1,PROD1,ERROR1(7),ERROR2(7),RAREA
        REAL PROD,RANGE,INTGRL,ERROR,ERRDB
        REAL WAREA,EAREA,AREA,ABE,ABA,EBE,EBA
        REAL FILTER,BPFO,BPF1
        REAL BCONST,CCONST,SIGMA,AL,ASIGMA
        REAL SIGMA0(7),SIGMA1(7)
        INTEGER I,II,J,K
        DATA H/3.6/
        DATA (W(I),I=1,20)/.07750594,.07703981,.07611036,.07472316,
&                          .07288658,.07061164,.06791204,.06480401,
&                          .06130624,.05743976,.05322784,.04869580,
&                          .04387090,.03878216,.03346019,.02793700,
&                          .02224584,.01642105,.01049828,.00452127/
        DATA (X(I),I=1,20)/.03877241,.11608407,.19269758,.26815218,
&                          .34199409,.41377920,.48307580,.54946712,
&                          .61255388,.67195668,.72731825,.77830565,
&                          .82461223,.86595950,.90209880,.93281280,
&                          .95791681,.97725994,.99072623,.99823770/
C
C     SYSTEM CONST FOR INTERNAL RANGE
        DATA RCONST/2.6/
```

```
C
C   CONSTANT TO CONVERT SLOPE IN DB SCALE INTO EXPONENTIAL SLOPE
      BBB=10.*ALOG10(EXP(1.))
      PI=4.*ATAN(1.0)
C
C   TWO WAY 3 DB BEAMWIDTHS IN DEGREES
      ABE=9.0
      ABA=9.0
C
      BE=ABE*PI/180.
      BA=ABA*PI/180.
C   TWO WAY EFFECTIVE BEAMWIDTHS IN DEGREES
      EBE=1.201*ABE
      EBA=1.201*ABA
C
C   INPUT SIGMAO AND OUTPUT THESE INPUT PARAMETERS
      DATA SIGMAO/-8.7,-17.,-26.6,-33.,-42.,-53.,-60./
      N=7
      CALL REGRES(SIGMAO,N,SLOPE,CORR)
      WRITE(6,699)
  699 FORMAT(/1X,'SIGMAO DATA, ORIGINAL AND UP TO 4TH REFINED')
      WRITE(6,698)
  698 FORMAT(/1X,'ANGLE    10      20      30      40      50      60
& 70   SLOPE CORRELATION')
      WRITE(6,697) (SIGMAO(I),I=1,7),SLOPE,CORR
  697 FORMAT(4X,7F7.2,1X,F7.4,2X,F7.4)
C
C
C    ASSIGN INTEGRATION CONSTS
      DO 100 I=21,40
        W(I)=W(I-20)
        X(I)=-X(I-20)
      100 CONTINUE
C
C   DO UNTIL 4 TH ITERATION
C
      DO 89 II=1,4
      SLOPP=-BBB/SLOPE
C
       DO 90 K=1,7
       ANGLE=FLOAT(K)*10.
         THEO=PI*ANGLE/180..
C
C
       RO=H/COS(THEO)
C
C    LIMITS UP TO FIRST SIDELOBES OF (SIN(X)/X)**4
       AL=PI*1.414/2.7831
       ALU=AL
       IF(K .EQ. 6 .OR. K .EQ. 7) ALU=1.5
       UPX=H*(TAN(THEO+ALU*BE))
       LOWX=H*(TAN(THEO-AL*BE))
       UPY=RO*TAN(AL*BA)
       LOWY=-UPY
       HALFX=(UPX-LOWX)/2.
       HALFY=(UPY-LOWY)/2.
       AVGX=(UPX+LOWX)/2.
       AVGY=(UPY+LOWY)/2.
C
       ISUM1=0.
```

```
      ISUM=0.
      DO 101 I=1,20
        YY=X(I)*HALFY+AVGY
        JSUM=0.
        JSUM1=0.
C
        DO 102 J=1,40
          XX=X(J)*HALFX+AVGX
          CALL GAIN1(XX,YY,H,THEO,G)
          RANGE=SQRT(XX**2+YY**2+H**2)
          FILTER=BPF1(RANGE,RO,RCONST)
          RANGE4=RANGE**4
          SIGMA=ASIGMA(XX,YY,H,ANGLE,SLOPP)
          PROD=G*FILTER*SIGMA/RANGE4
C
          PROD1=G*FILTER/RANGE4
          JSUM1=JSUM1+W(J)*PROD1
          JSUM=JSUM+W(J)*PROD
        102 CONTINUE
C
        ISUM=ISUM+W(I)*JSUM
        ISUM1=ISUM1+W(I)*JSUM1
      101 CONTINUE
C
      INTGRL=2.*HALFX*HALFY*ISUM1
C
C   WEIGHTED AREA USING 3DB BEAMWIDTHS WITHOUT SIGMA VARIATION
      WAREA=RO**4*INTGRL
C
C   WEIGHTED AREA INCLUDING SIGMA VARIATION
      RAREA=2.*HALFX*HALFY*ISUM*(RO**4)
C
C   EFFECTIVE AREA USING EFFECTIVE BEAMWIDTHS
C   AND CONST GAIN AND CONST RANGE
      EAREA=AREA(EBE,EBA,ANGLE,H,RCONST)
C
      ERROR1(K)=10.*ALOG10(WAREA/EAREA)
      ERROR2(K)=10.*ALOG10(RAREA/WAREA)
C
C      WRITE(6,999) ANGLE,ERROR1,ERROR2
C   999 FORMAT(1X,'ANGLE=',F5.1,2X,'ERROR1=',F6.3,' DB',2X,
C&                 'ERROR2=',F6.3,' DB')
C
C      WRITE(6,997) EAREA,WAREA,RAREA
C  997 FORMAT(12X,'EAREA=',F9.3,2X,'WAREA=',F9.3,2X,'RAREA=',F9.3)
C
   90 CONTINUE
C
      IF(II .NE. 1) GO TO 112
      DO 111 I=1,7
          SIGMA1(I)=SIGMA0(I)-ERROR1(I)
      111 CONTINUE
C
      CALL REGRES(SIGMA1,N,SLOPE,CORR)
      WRITE(6,697) (SIGMA1(I),I=1,7),SLOPE,CORR
      GO TO 114
   112 CONTINUE
C
      DO 113 I=1,7
         SIGMA0(I)=SIGMA1(I)-ERROR2(I)
```

```
  113 CONTINUE
C
      CALL REGRES(SIGMAD,N,SLOPE,CORR)
      WRITE(6,697) (SIGMAD(I),I=1,7),SLOPE,CORR
  114 CONTINUE
   89 CONTINUE
      STOP
      END
C
C****************
      SUBROUTINE GAIN1(X,Y,HH,TO,GG)
C
C GAUSSIAN ANTENNA GAIN FUNCTION
C  WITH TWO WAY BEAMWIDTHS=BE,BA
C
      COMMON BE,BA
      REAL BE,BA
      REAL X,Y,HH,TO,GG
      REAL THEA,PHIA,G1,G2,AA
C
      THEA=ATAN(X/HH)
      PHIA=ATAN(Y/(SQRT(X*X+HH*HH)))
      AA=4.*ALOG(2.)
      G1=EXP(-AA*((THEA-TO)**2)/(BE**2))
      G2=EXP(-AA*(PHIA**2)/(BA**2))
      GG=G1*G2
      RETURN
      END
C
C************************************
      SUBROUTINE GAIN2(X,Y,HH,TO,GG)
C
C  (SIN(X)/X)**4 TWO WAY ANTENNA GAIN PATTERN
C  WITH TWO WAY BEAMWIDTHS=BE,BA
C
      COMMON BE,BA
      REAL BE,BA
      REAL X,Y,HH,TO,GG
      REAL THEA,PHIA,G1,G2,BEONE,BAONE,A1,A2,G11,G22
C
C  ONE WAY 3DB BEAMWIDTHS
        BEONE=BE*SQRT(2.)
        BAONE=BA*SQRT(2.)
C
      THEA=ATAN(X/HH)
      PHIA=ATAN(Y/(SQRT(X*X+HH*HH)))
      A1=2.7831*(THEA-TO)/BEONE
      A2=2.7831*PHIA/BAONE
      G1=ABS((SIN(A1))/A1)
      IF(G1.LE. (1.0E-10)) GO TO 400
      G11=G1**4
      G2=ABS((SIN(A2))/A2)
      IF(G2 .LE. (1.0E-10)) GO TO 400
      G22=G2**4
      GG=G11*G22
      RETURN
C
  400 CONTINUE
      GG=0.
      RETURN
```

```
      END
C***********************
      REAL FUNCTION BPF1(R,RO,CONST)
C
C  TRAPEZOIDAL BANDPASS FILTER FUNCTION
C
      REAL R,RO,CONST
      REAL FIL,BW1,CF,LOWR,HIGHR,RRO,HF,LF
      DATA BW1,CF/12.0,50.0/
C
      FIL=BW1/(2.*CF)
      HF=1.+FIL
      LF=1.-FIL
      LOWR=RO*LF-FIL*CONST
      HIGHR=RO*HF+FIL*CONST
      RRO=(R+CONST)/(RO+CONST)
      IF(R.GT.HIGHR .AND. RRO.LE.(1.2*HF)) BPF1=10.**(-30.*(RRO-HF))
      IF(RRO.GT.(1.2*HF)) BPF1=0.
      IF(R.LE.HIGHR .AND. R.GE.LOWR) BPF1=1.
      IF(R.LT.LOWR) BPF1=10.**(145.*(RRO-LF)/4.)
      RETURN
      END
CC*******************
      REAL FUNCTION BPFO(R,RO,CONST)
C
C  RECTANGULAR BANDPASS FILTER FUNCTION
C
      REAL R,RO,CONST,FIL
      REAL BW,CF,LOWR,HIGHR
      DATA BW,CF/13.5,50.0/
C
      FIL=BW/(2.*CF)
      LOWR=RO*(1.-FIL)-CONST*FIL
      HIGHR=RO*(1.+FIL)+CONST*FIL
C
      IF(R.GE.LOWR .AND. R.LE.HIGHR) GOTO 800
      BPFO=0.
      RETURN
  800 CONTINUE
      BPFO=1.0
      RETURN
      END
CC*************
      REAL FUNCTION ASIGMA(X,Y,HH,ANGL,B)
C
C  THIS FUNCTION IS THE SIGMA VARIATION FUNCTION FOR POINT
C  P(X,Y) WITH POINTING ANGLE=ANGL AND SLOPE=B.
C
C
      REAL X,Y,HH,ANGL,B
      REAL THETA,ATHETA,PI
      PI=4.*ATAN(1.)
      THETA=ATAN(SQRT(X*X+Y*Y)/HH)
      ATHETA=THETA*180./PI
      ASIGMA=EXP(-(ATHETA-ANGL)/B)
      RETURN
      END
C***********************
      SUBROUTINE REGRES(Y,NN,SLOP,COR)
C
```

```
C     THIS SUBROUTINE CALCULATES THE SLOPE AND THE CORRELATION
C     COEFFICIENT OF AN ARRAY Y(NN). THE X-VARIABLE IS THE
C     INCIDENCE ANGLE FROM 10 TO 10*NN DEGREES.
C
        INTEGER NN
        REAL Y(NN),SLOP,SLOPE,COR,X(10),SY(10),SX(10)
        REAL SXY(10),AVG,AVX,AVY,BBB
C
        DO 10 I=1,NN
         X(I)=FLOAT(I)*10.
        10 CONTINUE
       NNN=NN
        AVX=AVG(X,NNN)
        AVY=AVG(Y,NNN)
        DO 11 I=1,NNN
          SY(I)=(Y(I)-AVY)**2
          SX(I)=(X(I)-AVX)**2
          SXY(I)=(X(I)-AVX)*(Y(I)-AVY)
       11 CONTINUE
        SLOP=AVG(SXY,NNN)/AVG(SX,NNN)
        COR=SLOP*SQRT(AVG(SX,NNN)/AVG(SY,NNN))
       RETURN
        END
C************
        REAL FUNCTION AVG(A,N)
C
C     THIS FUNCTION CALCULATES THE AVERAGE OF AN ARRAY WITH N
C     ELEMENTS.
C
        INTEGER N
        REAL A(N),SUM
        SUM=0.
        DO 12 I=1,N
          SUM=SUM+A(I)
        12 CONTINUE
        AVG=SUM/FLOAT(N)
        RETURN
        END
C*************
        REAL FUNCTION AREA(ELB,AZB,ANGL,H,RCONST)
C
C
C       THIS FUNCTION AREA CALCULATES THE EXACT ILLUMINATED AREA
C       FOR AN FM-CW RADAR. CALCULATION INCLUDES AREA LIMITING
C       BY THE BAND PASS FILTER .
C
C       INPUT PARAMETERS
C             ELB:ELEVATION BEAMWIDTH IN DEGREES
C             AZB:AZIMUTH BEAMWIDTH IN DEGREES
C             ANGL:INCIDENCE ANGLE IN DEGREE
C               H:HEIGHT OF THE ANTENNA
C             RCONST:SYSTEM CONST FOR INTERNAL RANGE
C
        REAL PI,H
        REAL ANGL,ELB,AZB,RCONST,ELBR,AZBR
        REAL THE,FC,BW,FIL,RE,RA
        REAL R,C,ALPA1,ALPA2,GAM,A,B,K,L1,L2,X1,X2
C
C       FUNCTIONS REFERENCED
C
```

```
      REAL LAREA1,LAREA2
C
      DATA FC,BW/50.0,13.5/
      PI=3.1415926
C
C  CONVERT BEAMWIDTHS TO RADIANS
C
      ELBR=PI*ELB/180.
      AZBR=PI*AZB/180.
C
      IF(IFIX(ANGL) .EQ. 0) GO TO 301
C
      THE=ANGL*PI/180.
      R=H/COS(THE)
      C=H*TAN(THE)
      FIL=BW/(2.*FC)
      ALPA1=R*(1.-FIL)-RCONST*FIL
      ALPA2=R*(1.+FIL)+RCONST*FIL
      GAM=H*(TAN(THE-ELBR/2.))
      RA=2.*R*(TAN(AZBR/2.))
      RE=H*(TAN(THE+ELBR/2.))-GAM
      A=RE/2.
      B=RA/2.
      K=GAM+A
      L1=C-GAM
      L2=RE-L1
      IF(ALPA1-H) 120,120,122
 120  X1=C
      GO TO 124
 122    X1=C-SQRT(ALPA1*ALPA1-H*H)
 124  CONTINUE
      X2=SQRT(ALPA2**2-H*H)-C
C
C  CHECK IF THERE IS ANY BEAM LIMITING DUE TO NARROW BPF
C  AND IF ANY, GO TO APPROPRIATE CASES
C
      IF(X1-L1) 200,201,201
 200  IF(X2-L2) 205,203,203
 201  IF(X2-L2) 204,202,202
C
 202  CONTINUE
C  CASE 1, NO LIMITING OF THE BEAM
      AREA=PI*A*B
      RETURN
C
 203  CONTINUE
C  CASE 2, NEAR SIDE OF THE BEAM IS LIMITED BY THE FILTER
      AREA=PI*A*B-LAREA1(A,B,C,K,X1)
      RETURN
C
 204  CONTINUE
C  CASE 3, FAR SIDE OF THE BEAM IS LIMITED BY THE FILTER
      AREA=LAREA2(A,B,C,K,X2)
      RETURN
C
 205  CONTINUE
C CASE 4, BOTH SIDES OF THE BEAM IS LIMITED BY THE FILTER
      AREA=LAREA2(A,B,C,K,X2)-LAREA1(A,B,C,K,X1)
      RETURN
C
```

```
  301 CONTINUE
      AREA=PI*H*H*TAN(ELBR/2.)*TAN(AZBR/2.)
      RETURN
      END
C
C ****************************************************
      REAL FUNCTION INTSEC(A1,A2,A3,A4)
C
C  THIS FUNCTION CALCULATES THE INTERSECTION OF THE ELLIPSE
C  (X-K)**2/A**2+Y**2/B**2=1 AND THE CIRCLE X**2+Y**2=A4**2
C
      REAL A1,A2,A3,A4
C  LOCAL VARIABLES
      REAL B1,B2,B3,B4,B5,B6
      B1=(A2+A1)*(A2-A1)
      B2=(A4+A2)*(A4-A2)
      B3=(A1*A2*A3)**2
      B4=A2*A2*A3
      B5=A1*A1
      B6=SQRT(B3-B5*B1*B2)
      INTSEC=(B4-B6)/B1
      RETURN
      END
C
C **************************************************
      REAL FUNCTION  MISC(THETA)
C
C  THIS FUNCTION CALCULATES (2*THETA-SIN(2*THETA))/4
C
      REAL THETA
      MISC=(2.*THETA-SIN(2.*THETA))/4.
      RETURN
      END
C
C *************************************************
      REAL FUNCTION THEX(MG,MP)
C
C  THIS FUNCTION CALCULATES  ARCTAN(SQRT(MG**2-MP**2)/MP)
C
      REAL MG,MP
      REAL MGP
      MGP=SQRT(MG*MG-MP*MP)
      THEX=ATAN(MGP/MP)
      RETURN
      END
C
C ***********************************************
      REAL FUNCTION LAREA1(AA,BB,CC,KK,XX1)
C
C  THIS FUNCTION CALCULATES THE NEAR-SIDE-BEAM LIMITED AREA
C  WHICH SHOULD BE SUBTRACTED FROM AREA CALCULATION
C
      REAL AA,BB,CC,KK,XX1
      REAL TTH1,IINT1,K<MP,TTHE2,IINT2,GG,PPSI
      REAL THEX,MISC,INTSEC
      GG=CC-XX1
      PPSI=INTSEC(AA,BB,KK,GG)
C
C  INTEGRATE ALONG THE CIRCLE
C
```

```
      TTH1=THEX(GG,PPSI)
      IINT1=GG*GG*MISC(TTH1)
C
C INTEGRATE ALONG THE ELLIPSE
C
      KKMP=KK-PPSI
      TTHE2=THEX(AA,KKMP)
      IINT2=AA*BB*MISC(TTHE2)
C
      LAREA1=2.*(IINT1+IINT2)
      RETURN
      END
C
C ***********************************************
      REAL FUNCTION LAREA2(AA,BB,CC,KK,XX2)
C
C  THIS FUNCTION CALCULATES THE AREA FOR THE CASE WHEN
C  THE FAR SIDE OF THE BEAM IS LIMITED BY THE FILTER
C
      REAL AA,BB,CC,KK,XX2
      REAL G,PSI,THE1,THE3,INT1,INT2,PI,KMP
      REAL THEX,MISC,INTSEC
      DATA PI/3.1415926/
C
      G=CC+XX2
      PSI=INTSEC(AA,BB,KK,G)
C
C  INTEGRAL FROM PSI TO G THROUGH THE CIRCLE
C
  THE1=THEX(G,PSI)
  INT1=G*G*MISC(THE1)
C
C  INTEGRAL FROM GAMMA TO PSI THROUGH THE ELLIPSE
C
      IF(KK-PSI) 300,301,302
 300  KMP=PSI-KK
      THE3=THEX(AA,KMP)
      INT2=AA*BB*(.5*PI-MISC(THE3))
      GO TO 303
 301  INT2=PI*AA*BB/4.
      GO TO 303
 302  THE3=THEX(AA,KK-PSI)
      INT2=AA*BB*MISC(THE3)
C
 303  LAREA2=2.*(INT1+INT2)
      RETURN
      END
```

END

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